Regularizing cubic open Neveu-Schwarz string field theory

JHEP11(2009)021

(http://iopscience.iop.org/1126-6708/2009/11/021)

Published by IOP Publishing for SISSA

Received: May 27, 2009
Revised: September 30, 2009
Accepted: October 22, 2009
Published: November 5, 2009

Regularizing cubic open Neveu-Schwarz string field theory

Nathan Berkovits[a,c] **and Warren Siegel**[b,c]

[a] *Instituto de Física Teórica, São Paulo State University, São Paulo, SP 01405-900, Brasil*

[b] *C.N. Yang Institute for Theoretical Physics, State University of New York, Stony Brook, NY 11794-3840, U.S.A.*

[c] *Kavli Institute for Theoretical Physics, University of California at Santa Barbara, CA 93106-4030, U.S.A.*

E-mail: nberkovi@ift.unesp.br, siegel@insti.physics.sunysb.edu

Abstract: After introducing non-minimal variables, the midpoint insertion of $Y\bar{Y}$ in cubic open Neveu-Schwarz string field theory can be replaced with an operator $\mathcal{N}_\rho$ depending on a constant parameter ρ. As in cubic open superstring field theory using the pure spinor formalism, the operator $\mathcal{N}_\rho$ is invertible and is equal to 1 up to a BRST-trivial quantity. So unlike the linearized equation of motion $Y\bar{Y}QV = 0$ which requires truncation of the Hilbert space in order to imply $QV = 0$, the linearized equation $\mathcal{N}_\rho QV = 0$ directly implies $QV = 0$.

Keywords: Superstrings and Heterotic Strings, String Field Theory

ArXiv ePrint: 0901.3386

 doi:10.1088/1126-6708/2009/11/021

Contents

1 Introduction

Open bosonic string field theory has recently been used to study classical solutions of string theory such as tachyon condensation that are difficult to analyze using first-quantized approaches. Some progress has been made in extending these techniques to open superstring field theory, and there are presently three versions of open superstring field theory available.

The first version is based on the cubic action [1, 2]

$$S_1 = \langle Y\bar{Y}(\frac{1}{2}VQV + \frac{1}{3}V * V * V)\rangle \tag{1.1}$$

where V is the Neveu-Schwarz (NS) string field of zero picture and $+1$ ghost-number in the small Hilbert space, and $Y\bar{Y}$ is an operator of -2 picture inserted at the string midpoint. The second version is based on the Wess-Zumino-Witten-like action [3]

$$S_2 = \langle (e^{-\phi}Qe^{\phi})(e^{-\phi}\eta e^{\phi}) + \int_0^1 dt\{(e^{-t\phi}Qe^{t\phi}),\ (e^{-t\phi}\eta e^{t\phi})\}(e^{-t\phi}\partial_t e^{t\phi})\rangle \tag{1.2}$$

where ϕ is the NS string field of zero picture and zero ghost-number in the large Hilbert space. And the third version is based on the cubic action [4]

$$S_3 = \langle \mathcal{N}_\rho\left(\frac{1}{2}\Phi Q\Phi + \frac{1}{3}\Phi * \Phi * \Phi\right)\rangle \tag{1.3}$$

where Φ is a superstring field of $+1$ ghost-number in the $GSO(+)$ sector using the non-minimal pure spinor formalism, and $\mathcal{N}_\rho = e^{-\rho\{Q,\chi\}}$ is a BRST-invariant regulator inserted at the midpoint which depends on a constant parameter ρ.

Each of these three versions has advantages and disadvantages. The first version of (1.1) has the advantage of being cubic, but has the disadvantage of being singular since the midpoint insertion $Y\bar{Y}$ is not invertible. So the linearized action $Y\bar{Y}QV = 0$ does not imply $QV = 0$ unless one truncates out states in the kernel of $Y\bar{Y}$. [5]

The second version of (1.2) has the disadvantage of being non-polynomial, but has the advantage of being non-singular since there are no midpoint insertions. So the linearized equation of motion is $\eta Q\phi = 0$, which implies $QV = 0$ where $V \equiv \eta\phi$ is in the small Hilbert space.

Finally, the third version of (1.3) has the advantage of being cubic and non-singular since, unlike the operator $Y\bar{Y}$ in (1.1), the operator $\mathcal{N}_\rho$ has no kernel and is invertible. So the linearized equation of motion $\mathcal{N}_\rho Q\Phi = 0$ implies $Q\Phi = 0$. The disadvantage of the third version is that Φ includes the $GSO(+)$ NS and Ramond sectors of the superstring, but does not include the $GSO(-)$ NS sector and cannot be used to describe tachyon condensation.

In this paper, we shall propose a fourth version of open superstring field theory which combines the advantages of the first and third versions and eliminates their disadvantages. After adding a pair of non-minimal variables to the NS formalism, it will be possible to replace the singular operator $Y\bar{Y}$ of the first version with a non-singular invertible operator $\mathcal{N}_\rho$ depending on the non-minimal variables and on a constant parameter ρ. The action will be

$$S_4 = \langle \mathcal{N}_\rho \left(\frac{1}{2} VQV + \frac{1}{3} V * V * V \right) \rangle \tag{1.4}$$

where V is a NS string field in the zero picture in the small Hilbert space which is allowed to depend on the non-minimal variables.

2 Cubic open NS string field theory

In the absence of operators involving $\delta(\gamma)$, functional integration over the bosonic (β,γ) ghost zero modes produces infinities in the NS tree amplitude. These delta functions can be inserted in a BRST-invariant manner using the inverse picture-changing operator

$$Y = c\partial\xi e^{-2\phi} \equiv c\frac{\delta(\gamma)}{\gamma} \tag{2.1}$$

where we use the notation

$$\gamma = \eta e^{\phi}, \quad \beta = \partial\xi e^{-\phi}, \quad \delta(\gamma) = e^{-\phi}, \quad \frac{\delta(\gamma)}{\gamma} = \partial\xi e^{-2\phi}. \tag{2.2}$$

Note that the OPE's of e^ϕ imply that $\delta(\gamma) = \gamma(\frac{\delta(\gamma)}{\gamma})$. Since γ has two zero modes on a disk, open string tree amplitudes require two inverse-picture-changing operators, and it is convenient to insert the operator $Y\bar{Y}$ at the midpoint interaction where $\bar{Y} = \bar{c}\bar{\partial}\bar{\xi}e^{-2\bar{\phi}}$ is constructed from the antiholomorphic ghosts.

Since $Y\bar{Y}$ has a non-trivial kernel, e.g. $Y\bar{Y}c = 0$, it is clear that $Y\bar{Y}$ is not invertible unless one truncates out states from the Hilbert space. In order to replace $Y\bar{Y}$ with an invertible operator, the first step will be to write

$$Y\bar{Y} = 4\int dr \int d\bar{r} \int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} d\bar{u}\ e^{rc+\bar{r}\bar{c}+u\gamma^2+\bar{u}\bar{\gamma}^2} \tag{2.3}$$

where r and $\bar{r}$ are fermionic variables and u and $\bar{u}$ are bosonic variables. Note that

$$4\int_{-\infty}^{\infty} du e^{u\gamma^2} \int_{-\infty}^{\infty} d\bar{u} e^{\bar{u}\bar{\gamma}^2} = 4\delta(\gamma^2)\delta(\bar{\gamma}^2) = \frac{\delta(\gamma)}{|\gamma|}\frac{\delta(\bar{\gamma})}{|\bar{\gamma}|} = \frac{\delta(\gamma)}{\gamma}\frac{\delta(\bar{\gamma})}{\bar{\gamma}}. \tag{2.4}$$

The next step will be to treat $(r,\bar{r})$ and $(u,\bar{u})$ as non-minimal worldsheet variables with conjugate momenta $(s,\bar{s})$ and $(v,\bar{v})$ by adding them to the worldsheet action

$$S = S_{\mathrm{RNS}} + \int d^2z(-s\bar{\partial}r - \bar{s}\partial\bar{r} + v\bar{\partial}u + \bar{v}\partial\bar{u}) \tag{2.5}$$

JHEP11(2009)021

and to the BRST operator

$$Q = Q_{\rm RNS} + \int dz\ vr + \int d\bar{z}\bar{v}\bar{r}. \tag{2.6}$$

Using the standard quartet argument, the additional terms in Q imply that physical states in the cohomology of Q are independent of the non-minimal variables. It will also be convenient to perform the similarity transformation

$$\begin{aligned} &Q \to e^{c(s\partial u+\frac{1}{2}\partial(su))+\bar{c}(\bar{s}\bar{\partial}\bar{u}+\frac{1}{2}\bar{\partial}(\bar{s}\bar{u}))}\ Q\ e^{-c(s\partial u+\frac{1}{2}\partial(su))-\bar{c}(\bar{s}\bar{\partial}\bar{u}+\frac{1}{2}\bar{\partial}(\bar{s}\bar{u}))} \\ &\quad = Q_{\rm RNS} + \int dz\Big[vr + c\Big(\frac{1}{2}\partial(vu) + v\partial u - \frac{1}{2}\partial(sr) - s\partial r\Big) + \gamma^2\Big(s\partial u + \frac{1}{2}\partial(su)\Big)\Big] \\ &\qquad\qquad + \int d\bar{z}\Big[\bar{v}\bar{r} + \bar{c}\Big(\frac{1}{2}\bar{\partial}(\bar{v}\bar{u}) + \bar{v}\bar{\partial}\bar{u} - \frac{1}{2}\bar{\partial}(\bar{s}\bar{r}) - \bar{s}\bar{\partial}\bar{r}\Big) + \bar{\gamma}^2\Big(\bar{s}\bar{\partial}\bar{u} + \frac{1}{2}\bar{\partial}(\bar{s}\bar{u})\Big)\Big] \end{aligned} \tag{2.7}$$

so that (u,v) and (r,s) each carry conformal weight $(-\frac{1}{2},\frac{3}{2})$.

The final step is to define

$$\mathcal{N}_\rho = e^{\rho\{Q,\chi\}} = e^{\rho[rc+\bar{r}\bar{c}+u(\gamma^2+\frac{3}{2}c\partial c)+\bar{u}(\bar{\gamma}^2+\frac{3}{2}\bar{c}\bar{\partial}\bar{c})]} \tag{2.8}$$

where $\chi = uc + \bar{u}\bar{c}$, ρ is a nonzero constant, and we have used that χ has $-\frac{3}{2}$ conformal weight to compute the $uc\partial c$ term in (2.8). Using a similar computation as in (2.3), it is easy to check that

$$4\int dr \int d\bar{r} \int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} d\bar{u}\mathcal{N}_\rho = Y\bar{Y} \tag{2.9}$$

where the ρ dependence cancels out and the $uc\partial c$ term in (2.8) does not contribute because of the factor of $c\bar{c}$ in $Y\bar{Y}$. Although the term in the exponential of (2.8) has zero picture, the right-hand side of (2.9) has picture -2 after integration over u and $\bar{u}$.

As in the regulator used in the non-minimal pure spinor formalism [4], on-shell amplitudes cannot depend on ρ since $\mathcal{N}_\rho = 1 + Q\Omega$ for some Ω. Although the amplitude is singular when $\rho = 0$, it is regularized at any non-zero value of ρ. And $\mathcal{N}_\rho QV = 0$ implies $QV = 0$ since (2.8) is easily inverted to $(\mathcal{N}_\rho)^{-1} = e^{-\rho\{Q,\chi\}}$.

To define a cubic open NS string field theory using $\mathcal{N}_\rho$, one needs to allow the string field V to depend both on the original NS worldsheet variables $(x^m, \psi^m; b, c, \beta, \gamma)$ and on the new non-minimal variables (r, s, u, v). Note that bosonization is unnecessary since both V and $\mathcal{N}_\rho$ can be expressed in terms of (β,γ) and $(\bar{\beta},\bar{\gamma})$. The cubic string field theory action is

$$S = \langle \mathcal{N}_\rho\Big(\frac{1}{2}VQV + \frac{1}{3}V * V * V\Big)\rangle \tag{2.10}$$

where $\langle\ldots\rangle$ is defined as usual by functional integration over all the worldsheet variables.

Since u and r have two zero modes on the disk, their zero mode integration reproduces the operator of -2 picture in (2.9). So if one writes $V = V_0 + \widetilde{V}$ where V_0 is independent of the non-minimal variables, (2.9) implies that the terms in S which are independent of $\widetilde{V}$ are the same as in the original cubic action of (1.1). However, the terms in S which depend on $\widetilde{V}$ are necessary for guaranteeing that the linearized equation of motion is equivalent to $QV = 0$.

To see how $\widetilde{V}$ contributes to the action, note that rescaling

$$u \to \frac{u}{\rho}, \qquad r \to \frac{r}{\rho}, \qquad \bar{u} \to \frac{\bar{u}}{\rho}, \qquad \bar{r} \to \frac{\bar{r}}{\rho}, \tag{2.11}$$

$$v \to v\rho, \qquad s \to s\rho, \qquad \bar{v} \to \bar{v}\rho, \qquad \bar{s} \to \bar{s}\bar{\rho}, \tag{2.12}$$

removes the ρ dependence from $\mathcal{N}_\rho$ and leaves invariant the worldsheet action and BRST operator. After this rescaling, the string field depends on ρ as

$$V = \sum_{n=-\infty}^{\infty} \rho^{-n} V_n \tag{2.13}$$

where n counts the number of non-minimal fields in V_n, i.e.

$$[\int dz(uv + rs) + \int d\bar{z}(\bar{u}\bar{v} + \bar{r}\bar{s})\,,\ V_n] = nV_n. \tag{2.14}$$

Note that for string fields of finite conformal weight, n is bounded from below since $(v, s, \bar{v}, \bar{s})$ carry positive conformal weight.

Using (2.13), the action of (2.10) can be expressed as $S = \sum_{n=-\infty}^{\infty} S_n \rho^{-n}$ where

$$S_n = \langle \mathcal{N}_{\rho=1} \left(\frac{1}{2} \sum_{m=-\infty}^{\infty} V_{n-m} Q V_m + \frac{1}{3} \sum_{m,p=-\infty}^{\infty} V_{n-m-p} * V_m * V_p \right) \rangle. \tag{2.15}$$

As in the cubic action of (1.3) using the pure spinor formalism, (2.15) involves an infinite chain of auxiliary fields V_n depending on the non-minimal variables. Since the non-minimal variables include bosons, (2.13) and (2.15) resemble the construction of superfields and actions in harmonic superspace. It should be noted that, unlike the non-minimal variables in the pure spinor formalism which all carry non-negative conformal weight, the non-minimal variables u and r carry $-\frac{1}{2}$ conformal weight. So V_n for $n > 0$ involves states of negative conformal weight which could complicate computations using level truncation.

A related difficulty that has recently been discussed by Kroyter in [6] is caused by the nonzero conformal weight of $\chi = uc + \bar{u}\bar{c}$. Since the map of the cubic vertex to the disk is singular at the midpoint, any midpoint operator of nonzero conformal weight will induce a singularity in evaluating correlation functions on the disk. Fortunately, this problem appears to have been resolved in [6] by introducing a second quartet of non-minimal fields, $(\widetilde{u}, \widetilde{v}, \widetilde{r}, \widetilde{s})$ which all have conformal dimension $\frac{1}{2}$, and suitably modifying the BRST operator to include these new non-minimal fields. The operator $\chi = uc + \bar{u}\bar{c}$ can then be replaced by $\widetilde{\chi} = (\widetilde{u})^3 uc + (\widetilde{\bar{u}})^3 \bar{u}\bar{c}$ which now carries zero conformal weight. Since the new non-minimal fields have no zero modes on the disk and decouple from the cohomology, it is trivial to integrate them out. And since the midpoint operator now carries zero conformal weight, there are no longer singularities in correlation functions on the disk.

Another possible difficulty, which is also a difficulty with all the other cubic superstring field theory actions, is gauge-fixing. Since the midpoint insertion of $\mathcal{N}_\rho$ (like the insertion of $Y\bar{Y}$) involves the c ghost, fixing the $b_0 = 0$ gauge may be subtle. One could try to implement alternative gauge choices such as Schnabl gauge or the gauge choices of [1], but these also have subtleties. Note that in the cubic action using the pure spinor formalism, there are difficulties with gauge-fixing because of the $(\lambda\hat{\lambda})$ poles in the b ghost [4]. The only action that appears to be free of gauge-fixing difficulties is the Wess-Zumino-Witten-like action of (1.2) where one can easily choose the gauge-fixing conditions $b_0 = \xi_0 = 0$ as in [7].

It would be interesting to extend the action of this paper to include the Ramond sector. Although one can describe the $GSO(+)$ Ramond sector using the pure spinor version of (1.3), there is no non-singular action which can covariantly describe both the $GSO(+)$ and $GSO(-)$ Ramond and NS sectors. It is intriguing that the non-minimal worldsheet fields (u, v, r, s) introduced here have the same statistics and conformal weights as the non-minimal worldsheet fields $(\widetilde{\gamma}, \widetilde{\beta}, \xi, \mu)$ which were used in [8] to allow a more symmetric treatment of the NS and Ramond sectors.

Acknowledgments

NB and WS would like to thank Michael Kroyter for valuable discussions and the KITP conference "Fundamental Aspects of Superstring Theory" where this work was done. This research was supported in part by the National Science Foundation under Grant. No. PHY05-51164. The research of NB was also partially supported by CNPq grant 300256/94-9 and FAPESP grant 04/11426-0, and the reserch of WS was partially supported by National Science Foundation Grant. No. PHY-0653342.

References

[1] C.R. Preitschopf, C.B. Thorn and S.A. Yost, *Superstring field theory*, *Nucl. Phys.* **B 337** (1990) 363 [SPIRES].

[2] I.Y. Arefeva and P.B. Medvedev, *Truncation, picture changing operation and space time supersymmetry in Neveu-Schwarz-Ramond string field theory*, *Phys. Lett.* **B 202** (1988) 510 [SPIRES].

[3] N. Berkovits, *Super-Poincaré invariant superstring field theory*, *Nucl. Phys.* **B 450** (1995) 90 [hep-th/9503099] [SPIRES].

[4] N. Berkovits, *Pure spinor formalism as an $N = 2$ topological string*, *JHEP* **10** (2005) 089 [hep-th/0509120] [SPIRES].

[5] I.Y. Arefeva and P.B. Medvedev, *Anomalies in Witten's field theory ofl the NSR string*, *Phys. Lett.* **B 212** (1988) 299 [SPIRES].

[6] M. Kroyter, *On string fields and superstring field theories*, *JHEP* **08** (2009) 044 [arXiv:0905.1170] [SPIRES].

[7] N. Berkovits, A. Sen and B. Zwiebach, *Tachyon condensation in superstring field theory*, *Nucl. Phys.* **B 587** (2000) 147 [hep-th/0002211] [SPIRES].

[8] N. Berkovits, M.T. Hatsuda and W. Siegel, *The big picture*, *Nucl. Phys.* **B 371** (1992) 434 [hep-th/9108021] [SPIRES].

JHEP11(2009)021